AIAA JOURNAL
Vol. 42, No. 1, January 2004

Fuzzy Approach to the Mechanics of Fiber-Reinforced Composite Materials

Singiresu S. Rao* and Qing Liu†
University of Miami, Coral Gables, Florida 33124

Fiber-reinforced composite materials have been used very widely and have emerged as a major class of structural materials in recent years. They are either used or being considered for use as substitutes for metals in many weight-critical components in aerospace and other industries. In most practical applications, the material parameters vary considerably and are subject to uncertainties, mainly due to the uncontrollable aspects associated with the manufacturing process. The probabilistic methods cannot be applied easily because the exact probability distributions of the uncertain parameters are not known. Also, in some situations, the parameters of a composite material are known only in linguistic form. Hence, fuzzy concepts are introduced to model the uncertainties encountered in composite materials, which may be described in uncertain terms or in imprecise/linguistic form as fuzzy parameters. When the basic fuzzy arithmetic computations, such as fuzzy addition, subtraction, multiplication, and division, and fuzzy square root and fuzzy trigonometry computations are used, different composite material mechanics expressions are explored, and the fuzzy analysis of fiber-reinforced composite material mechanics is presented. Numerical examples are given to demonstrate the feasibility and applicability of the approach. Other types of uncertainties, such as interval and probabilistic parameters, can also be accommodated with the approach.

Nomenclature

A	=	cross-sectional area
E	=	Young's modulus
E_{11} (E_{xx})	=	longitudinal modulus (along x direction) of a composite material
E_{22} (E_{yy})	=	transverse modulus (along y direction) of a composite material
F	=	fuzzy set
F_α, F^α	=	ordinary subset of F corresponding to level α
$F_\alpha^-(F_1^\alpha)$	=	lower bound of a fuzzy number F at the α level
$F_\alpha^+(F_2^\alpha)$	=	upper bound of a fuzzy number F at the α level
G, G_{ij}	=	shear modulus, shear modulus in plane ij
L	=	fiber length
$\max(X, Y)$ or $X \vee Y$	=	maximum of X and Y
$\min(X, Y)$ or $X \wedge Y$	=	minimum of X and Y
m_x	=	influence of shear stresses on extensional strains
m_y	=	influence of normal stresses on shear strains
P	=	load
R	=	radius
$\boldsymbol{R}(\boldsymbol{R}^+)$	=	set of real (nonnegative real) numbers
U	=	universe set
V	=	volume fraction
α	=	α cut, level α of a membership function
α_{ij}	=	coefficients of thermal linear expansion where $i = j$ along ii direction, or shear expansion where $i \neq j$
ε	=	linear (normal) strain
θ	=	lamina orientation angle
λ	=	number between 0 and 1
$\mu_F(x)$	=	membership function for the element x with respect to the fuzzy subset F (level of presumption)
ν	=	Poisson's ratio
ν_{12}	=	major Poisson's ratio of a composite material
ν_{21}	=	minor Poisson's ratio of a composite material
σ, σ_{ii}	=	normal stress, normal stress along direction i
τ	=	shear stress
$(+)$	=	fuzzy number addition by max–min convolution
$(-)$	=	fuzzy number subtraction by max–min convolution
$(\cdot)$	=	fuzzy number multiplication by max–min convolution
$(/)$	=	fuzzy number division by max–min convolution
$(\wedge)$, $(\vee)$	=	minimum (maximum) of fuzzy numbers by max-min convolution

Subscripts

C	=	composite material
F	=	fiber of the composite material
fu	=	fiber ultimate
ltu	=	longitudinal tensile unidirectional
m	=	matrix of the composite material

I. Introduction

ALTHOUGH fuzzy analysis and the theories of composite material mechanics have been developed during the past several years, the area of fuzzy analysis of composite material mechanics remains unexplored. For fiber-reinforced composite materials, the various parameters, such as the elastic modulus and fiber volume fraction, are not precisely known, and the parameter information may be vague, imprecise, qualitative, linguistic, or incomplete. This is mainly due to the complex processes and the human judgment involved. During the manufacturing process, a number of factors, including resin chemistry, cure temperature and pressure, catalyst reactivity, degree of cure, viscosity of the polymer melts, the curing

Received 24 August 2002; revision received 28 July 2003; accepted for publication 5 August 2003.

*Professor and Chairman, Department of Mechanical Engineering. Associate Fellow AIAA.

†Graduate Student, Department of Mechanical Engineering.

characteristics of the resin–catalyst combination, shrinkage caused by curing as well as thermal contraction, and presence of voids, will affect the final characteristics of the composite material. Some of these factors, such as the degree of cure, curing characteristics of the resin–catalyst combination, and process time, can only be experimentally determined and, hence, are not precisely known. Similarly the desired fiber orientation angles, for example, in the plies, cannot be achieved accurately due to the machine tolerances involved. Thus, most of the parameters in a composite material are imprecise and are subject to uncertainties. Information such as Young's modulus of the material is within the range of 3.5–3.6 GPa, fiber volume fraction is about 5%, and fiber length is slightly larger than 10 mm cannot be handled successfully by deterministic and probabilistic approaches. If approximate values of the parameters are used in finding the material characteristics and response parameters of the composite material, the accuracy and reliability of the results cannot be assured. The interval analysis methods can be used if only the ranges of the uncertain parameters are known. The probabilistic approach can be used if the exact probability distributions of the uncertain parameters are known, in many cases this might require considerable amount of data that is either impossible or unrealistic to obtain. The fuzzy set theory can be used conveniently to model the uncertainties of composite materials. By the use of fuzzy concepts, the mechanics of fiber-reinforced composite material can be analyzed more realistically and accurately. Because a fuzzy variable is usually described by a membership function, or a fuzzy set, it is possible to locate it inside a closed interval of $\boldsymbol{R}$, an interval of confidence of $\boldsymbol{R}$: $X_\alpha = [a_1^\alpha, a_2^\alpha]$ for each α level, and $\alpha \in [0, 1]$, with $a_1^\alpha \leq a_2^\alpha$ (Ref. 1). Each parameter of a composite material can be treated as a fuzzy variable. Therefore, all of the computations of material mechanics can be performed in fuzzy form. The membership function of a fuzzy set is usually described by a function that maps a universe of discourse onto the unit interval [0, 1], which is the coupling between the level α ($\alpha \in [0, 1]$) of presumption and the interval of confidence at level α; thus, the fuzzy computational results related to composite material mechanics will also fall in the same range. In this work, the parameters of composite material are treated as fuzzy variables, and the analyses are based on fuzzy computations. For the purpose of fuzzy computation, all variables are defined on sets that are combined in the same universe of discourse.

II. Reasons for Using the Fuzzy Approach

First, the deterministic approach imposes severe limitations on composites due to the higher scatter in strength characteristics and additional potential failure modes. In fact, if all of the various design allowable knockdown factors are considered in a deterministic process, the carbon/epoxy material will have virtually no specific strength advantage over aluminum.[2] The probabilistic approach offers the potential to quantify more accurately the scatter in composite material behavior. The manufacturers of composite materials usually conduct tests of basic constituent properties, and the results of tests form a good database. An analysis of these databases indicate that it is not always possible to fit well-defined probability distributions to the basic constituent properties such as fiber modulus, fiber longitudinal tensile strength, ply thickness, and unidirectional laminate strength. For example, the distribution of the diameter and the variability of the tensile strength of alumina fibers produced with a nominal diameter of 2.8 μm is shown in Fig. 1 (Ref. 3).

It can be seen that no probability distribution can represent the observed distribution of the fiber diameters. However, a fuzzy set with specific preference values for different values in the observed range of diameters can be used to model the behavior more accurately.

Second, depending on the nature of imprecision and uncertainty present, the analysis/design of the system can be conducted using the probabilistic approach, internal analysis, or fuzzy theory. In the probabilistic approach, the uncertain parameters are treated as random variables following specific probability distributions. In the interval analysis, simple ranges are assumed for the parameters. In addition to the range, if a preference function is used to describe the desirability (occurrence) of different values within the range, fuzzy theory can be used. The fuzzy approach can also be used when

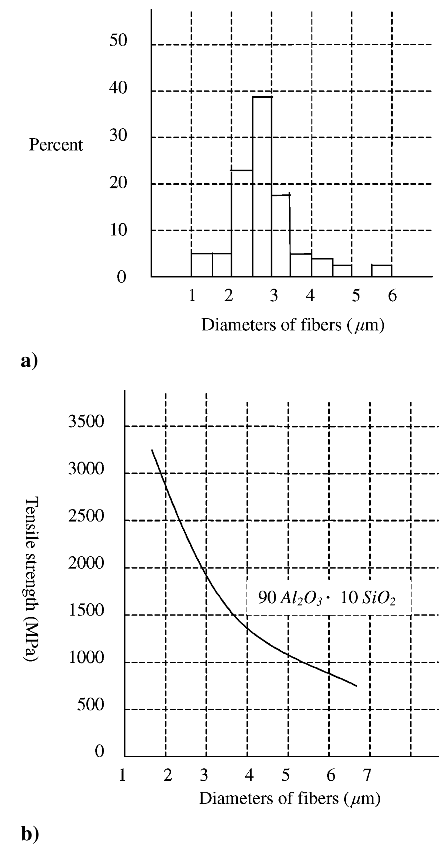

Fig. 1 Typical distribution of diameter and variation of tensile strength of alumina fibers of nominal diameter 2.8 μm (Ref. 3).

the uncertain parameters are described in a qualitative or linguistic form. The fuzzy approach can be considered, in some sense, as the most general type of uncertainty analysis. For example, if only the ranges of the uncertain parameters are available, the fuzzy analysis, with a membership function (α cut) of zero, can be used to predict the response of the system. Similarly, if the distribution functions of the random parameters are known (even with some imprecision), the fuzzy analysis can be used by constructing suitable membership functions that correspond to the probability distributions. The reason is that several different shapes with positive (convex), negative (concave), or zero (linear) values of the coefficient of membership satiation can be used for modeling different types of imprecision. Although not all fuzzy quantities have statistical basis for defining their membership functions, the membership functions of a fuzzy set can be based on the available statistical or probabilistic data.[4] If statistical data are known, the membership function μ (x) can be determined as

$$\mu(x) = \lambda p(x) \tag{1}$$

where

$$\lambda = 1/\max\{p(x)\} \tag{2}$$

and $p(x)$ is the probability mass/density function or its estimate derived from the histogram of the feature (X) used in defining the fuzzy set. Equation (2) satisfies the possibility–probability consistency principle that can be stated as follows: The degree of possibility of an element is greater than or equal to its degree of probability. If the membership function is used as a grade for possibility, the consistency principle can be restated as

$$\max_{x \in D} \left\{ \frac{\mu(x)}{\max \mu(x)} \right\} \geq \int_D p(x)\, \mathrm{d}x \tag{3}$$

for any set D on the real line. In some cases, the probability density function can be used to describe the membership function by

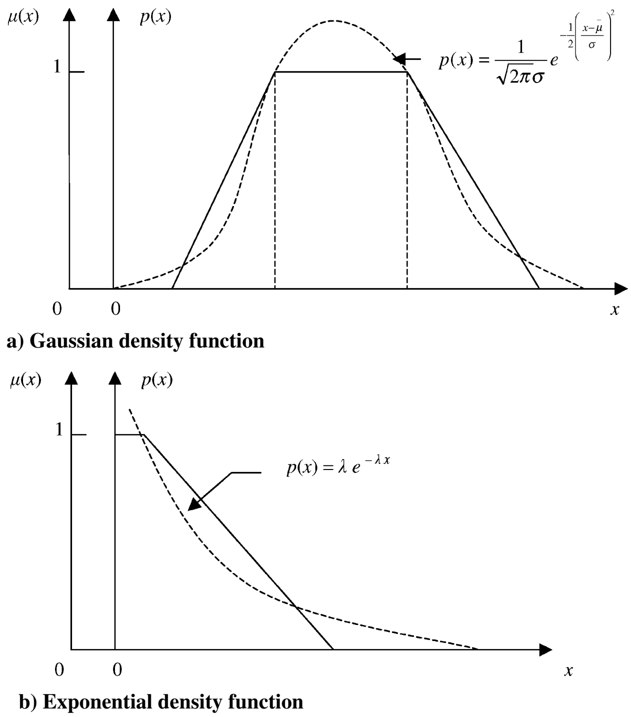

Fig. 2 Approximation of probability distributions as fuzzy sets.

preserving the overall shape of the density function. This is indicated for the Gaussian density function in Fig. 2a and the exponential density function in Fig. 2b.

Third, the current practice is to give the uncertain data on composite materials in terms of statistical variations. Although classical statistical/probabilistic methods can be used in such situations, the results may not be very accurate because the given statistical variations of input data are subject to certain confidence level(s). In such cases, fuzzy theory can provide equally meaningful results. In fact, fuzzy theory can also handle statistical variations in many cases. As stated earlier, fuzzy theory can be considered as a generalization of statistical/probability theory in some sense. The linguistic description of composite material properties may not be commonly used in current practice. However, the fuzzy theory and the computational aspects presented in this paper can be used in case such a situation arises. For example, if there is damage or failure in a structural system such as an aircraft or a space shuttle, and if the load and material conditions that led to the failure are not known precisely, fuzzy theory can be applied to draw certain inferences and, possibly, some meaningful conclusions.

III. Basic Fuzzy Operations

A fuzzy set can be represented as

$$F=\{(x,\mu_F(x))|x\in U\},\qquad \text{where}\quad \alpha=\mu_F:U\to[0,1] \tag{4}$$

A fuzzy number is defined as a normal and convex fuzzy set, where a fuzzy set F is convex if and only if $\forall x^{(1)}, x^{(2)}\in R$, $\forall\lambda\in[0,1]$,

$$\mu_F\left[\lambda x^{(1)}+(1-\lambda)x^{(2)}\right]\ge\mu_F\left(x^{(1)}\right)\wedge\mu_F\left(x^{(2)}\right) \tag{5}$$

where $X\wedge Y$ means minimum of X and Y and a fuzzy set is normal if and only if

$$\forall x\in R:\bigvee_n\mu_F(x)=1 \tag{6}$$

where $X\vee Y$ means maximum of X and Y and n is the total number of membership functions of F. This means that the highest value of $\mu_F(x)$is equal to 1. This maximum may or may not be unique. If A and B are two fuzzy numbers, their union is defined as the set whose members have the maximum of the two membership values, $\mu_{A\cup B}=\mu_A\vee\mu_B$, and the intersection operation chooses the minimum of the two membership values, $\mu_{A\cap B}=\mu_A\wedge\mu_B$.

Fuzzy arithmetic operations include fuzzy addition, fuzzy subtraction, fuzzy multiplication, and fuzzy division. In this work, fuzzy arithmetic operations are denoted as (**), where ** represents deterministic arithmetic operations such addition, subtraction, multiplication, and division. Thus, + denotes the deterministic addition, whereas (+) represents the fuzzy addition, similarly, x indicates a deterministic number, whereas (x) denotes a fuzzy number. Fuzzy arithmetic operations have features that are different from those of deterministic arithmetic. Fuzzy addition and fuzzy multiplication are commutative, associative, and distributive, but neither fuzzy subtraction nor fuzzy division is associative because $A(-)B(+)B\neq A$, and $[A(/)B](\cdot)B\neq A$. Also, a fuzzy zero (0) is defined as a fuzzy number in which the value zero has a membership value of one, and the left and right numbers of zero may not be the same. Similarly, a fuzzy one (1) is defined as a fuzzy number in which the value one has a membership value of one, and the left and right numbers of one may not be the same. The fuzzy arithmetic operation of two fuzzy numbers A and B is

$$A(^{**})B=\mu_{A(^{**})B}(z)=\bigvee_{z=x^{**}y}[\mu_A(x)\wedge\mu_B(y)] \tag{7}$$

which can also be expressed as

$$A_\alpha(+)B_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right](+)\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$$
$$=\left[a_1^{(\alpha)}+b_1^{(\alpha)},a_2^{(\alpha)}+b_2^{(\alpha)}\right]$$
$$A_\alpha(-)B_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right](-)\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$$
$$=\left[a_1^{(\alpha)}-b_2^{(\alpha)},a_2^{(\alpha)}-b_1^{(\alpha)}\right]$$
$$A_\alpha(\cdot)B_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right](\cdot)\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$$
$$=\left[a_1^{(\alpha)}\cdot b_1^{(\alpha)},a_2^{(\alpha)}\cdot b_2^{(\alpha)}\right]$$
$$A_\alpha(/)B_\alpha=\left[a_1^{(\alpha)},a_2^{(\alpha)}\right](/)\left[b_1^{(\alpha)},b_2^{(\alpha)}\right]$$
$$=\left[a_1^{(\alpha)}/b_2^{(\alpha)},a_2^{(\alpha)}/b_1^{(\alpha)}\right],\qquad 0\notin\left[b_1^{(\alpha)},b_2^{(\alpha)}\right] \tag{8}$$

where A_α and B_α are the intervals of confidence of A and B, respectively, for the level of presumption α, $\alpha\in[0,1]$, and $A_\alpha=\{x|\mu_A(x)\ge\alpha\}$ and $B_\alpha=\{x|\mu_B(x)\ge\alpha\}$.

Powers of fuzzy number are computed by repetitive multiplication operations. Summation of fuzzy numbers is computed by repetitive additions. Fuzzy root (fuzzy inverse, fuzzy absolute value), is evaluated by taking the root (reciprocal value, absolute value) of each element of the fuzzy number, without changing the associated membership values. For trigonometry operations, for $X_\alpha=[x_1^\alpha,x_2^\alpha]$, the following equations are defined:

$$\cos(X_\alpha)=\left[\cos\left(x_2^\alpha\right),\cos\left(x_1^\alpha\right)\right],\qquad x_1^\alpha,x_2^\alpha\in[0,\pi/2]$$
$$\sin(X_\alpha)=\left[\sin\left(x_1^\alpha\right),\sin\left(x_2^\alpha\right)\right],\qquad x_1^\alpha,x_2^\alpha\in[0,\pi/2] \tag{9}$$
$$\cosh(X_\alpha)=\left[\cosh\left(x_1^\alpha\right),\cosh\left(x_2^\alpha\right)\right],\qquad x_1^\alpha,x_2^\alpha\in R^+$$
$$\sinh(X_\alpha)=\left[\sinh\left(x_1^\alpha\right),\sinh\left(x_2^\alpha\right)\right],\qquad x_1^\alpha,x_2^\alpha\in R \tag{10}$$

If other quadrants are to be used, it is necessary to develop the relevant expressions. If one of the operands is a deterministic number $k\in\boldsymbol{R}^+$, and the other one is a fuzzy number $X_\alpha=[x_1^\alpha,x_2^\alpha]$, then the multiplication result will be

$$k(\cdot)X=[k,k](\cdot)\left[x_1^\alpha,x_2^\alpha\right]=\left[k\cdot x_1^\alpha,k\cdot x_2^\alpha\right] \tag{11}$$

Fuzzy sets can also be used to manipulate linguistic variables.[1,5] The linguistic variables, including labels such as small, big, low, and high, hedges such as very, quite, and extremely, negation (not),

Table 1 Fuzzy representation of typical linguistic statements

Set	Linguistic statement	Fiber content, x: 1	2	5	7	9	11
A	Low	1.0	1.0	0.8	0.5	0.3	0.0
$\bar{A}$	Not low	0.0	0.0	0.2	0.5	0.7	1.0
A^2	Very low	1.0	1.0	0.64	0.25	0.09	0.0
A^4	Very very low	1.0	1.0	0.4096	0.0625	0.0081	0.0
$\bar{A}^2$	Not very low	0.0	0.0	0.36	0.75	0.91	1.0
B	High	0.0	0.2	0.4	0.7	1.0	1.0
B^2	Very high	0.0	0.04	0.16	0.49	1.0	1.0
$A^2 \cup B^2$	Very low or very high	1.0	1.0	0.64	0.49	1.0	1.0
$A \cap \bar{A}^2$	Low but not very low	0.0	0.0	0.36	0.5	0.3	0.0

and connectives (and, but, or), can be assembled into relatively complex statements such as very low or not very high, and their fuzzy representations can be compounded with the operations indicated earlier. Fuzzy hedge operations differ from arithmetic operations because they do not affect the values contained within a fuzzy number. A hedge operates only on the membership function (power of the membership function of A, $\mu_{\text{hedge}}(x) = [\mu_A(x)]^y$, where y is a positive real number) of the fuzzy number A. When y is less than one, it is called concentration, and when y is greater than one, it is called dilation. For example, low is a fuzzy set in which all values less than a certain number are given the membership value of one, and high is a fuzzy set in which all values greater than a certain number are given the membership value of one. The fuzzy representation of typical linguistic statements associated with low and high are shown in Table 1. In this work, fuzzy analysis of the characteristics of a unidirectional lamina is considered.

IV. Longitudinal Tensile Loading in a Unidirectional Lamina

A. Continuous Parallel Fibers

Let the Young's moduli of fibers and matrix E_f and E_m be two triangular fuzzy numbers. In addition, let the fiber volume fraction $v_f = A_f/A_c$ and the matrix volume fraction $v_m = A_m/A_c$ also be fuzzy numbers, where A_f is the net cross-sectional area of the fibers, A_m is the net cross-sectional area of the matrix, and $A_c = A_f\ (+)\ A_m$. This implies that $v_m\ (+)\ v_f = (1)$. When the bonding between fibers and matrix is perfect,

$$\varepsilon_f = \varepsilon_m = \varepsilon_c \tag{12}$$

where ε_f, ε_m, and ε_c are the longitudinal strains in fibers, matrix, and composite, respectively, and are assumed to be deterministic to assure perfect bonding. Both fibers and matrix are assumed to be elastic with E_f, E_m, v_f, and v_m as fuzzy. From the level α of presumption, the longitudinal stresses in the fiber, the matrix, and the composite can be calculated as[6]

$$\sigma_{f_\alpha} = E_{f_\alpha}(\cdot)\varepsilon_c = [\varepsilon_c, \varepsilon_c](\cdot)\left[E_{f_1}^\alpha, E_{f_2}^\alpha\right] = \left[\varepsilon_c E_{f_1}^\alpha, \varepsilon_c E_{f_2}^\alpha\right] \tag{13}$$

$$\sigma_{m_\alpha} = E_{m_\alpha}(\cdot)\varepsilon_c = [\varepsilon_c, \varepsilon_c](\cdot)\left[E_{m_1}^\alpha, E_{m_2}^\alpha\right] = \left[\varepsilon_c E_{m_1}^\alpha, \varepsilon_c E_{m_2}^\alpha\right] \tag{14}$$

$$\begin{aligned} \sigma_{c_\alpha} &= \sigma_{f_\alpha}(\cdot)v_{f_\alpha}(+)\sigma_{m_\alpha}(\cdot)v_{m_\alpha} \\ &= \left[\sigma_{f_1}^\alpha, \sigma_{f_2}^\alpha\right](\cdot)\left[v_{f_1}^\alpha, v_{f_2}^\alpha\right](+)\left[\sigma_{m_1}^\alpha, \sigma_{m_2}^\alpha\right](\cdot)\left[v_{m_1}^\alpha, v_{m_2}^\alpha\right] \\ &= \left[\sigma_{f_1}^\alpha v_{f_1}^\alpha + \sigma_{m_1}^\alpha v_{m_1}^\alpha, \sigma_{f_2}^\alpha v_{f_2}^\alpha + \sigma_{m_2}^\alpha v_{m_2}^\alpha\right] \end{aligned} \tag{15}$$

Equation (15) can also be written as

$$\mu_{(\sigma_f(\cdot)v_f)(+)(\sigma_m(\cdot)v_m)}(z) = \bigvee_{z=x+y}\left[\mu_{\sigma_f(\cdot)v_f}(x) \wedge \mu_{\sigma_m(\cdot)v_m}(y)\right]$$

$$= \bigvee_{z=x+y}\left(\left\{\bigvee_{x=x_f\cdot y_f}\left[\mu_{\sigma_f}(x_f) \wedge \mu_{v_f}(y_f)\right]\right\} \wedge \left\{\bigvee_{y=x_m\cdot y_m}\left[\mu_{\sigma_m}(x_m) \wedge \mu_{v_m}(y_m)\right]\right\}\right) \tag{16}$$

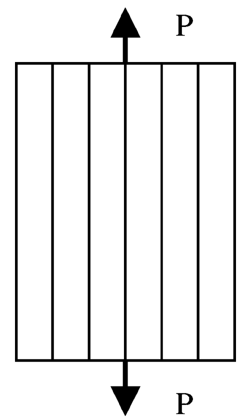

Fig. 3 Continuous parallel fiber lamina.

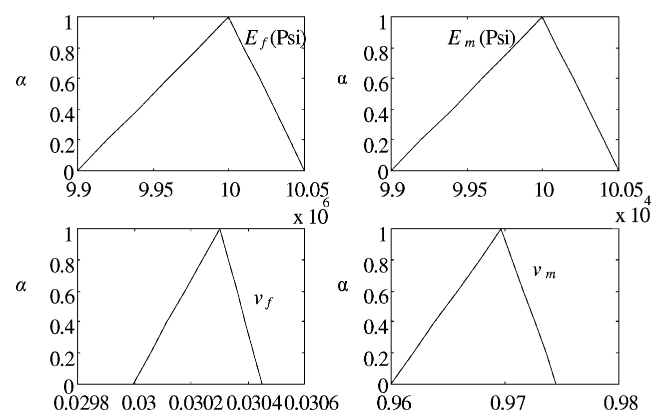

Fig. 4 Membership functions of E_f, E_m, v_m, and v_f.

where $\mu_A(x) \in [0, 1]$ is the membership function of the element x with respect to the fuzzy subset A (level of presumption), and σ_f, σ_m, and σ_c are the average tensile stresses in the fibers, matrix, and composite, respectively. Figure 3 shows a continuous parallel fiber lamina.

The lamina is composed of an epoxy matrix and E–glass fibers, and the fuzzy parameters, in triangular form, are shown in Fig. 4. Figure 5 shows the stresses given by Eqs. (13–15) for a continuous parallel fiber lamina under a longitudinal tensile loading.

Because uncertainties are present in all of the input parameters, the final stresses are also expected to have uncertainties, each defined by a range. For each strain value, the stress becomes a fuzzy number. Figure 6 shows the graphical representation of the stresses corresponding to $\varepsilon = 0.025$. It can be seen that the membership functions of the stresses may not be triangular although the membership function of each of the input parameters is assumed to be of triangular form. When both sides of Eq. (15) are divided by ε_c, the longitudinal modulus of the composite can be expressed as

$$\begin{aligned} E_{L_\alpha} &= E_{f_\alpha}(\cdot)\ v_{f_\alpha}(+)E_{m_\alpha}(\cdot)v_{m_\alpha} \\ &= \left[E_{f_1}^\alpha v_{f_1}^\alpha + E_{m_1}^\alpha v_{m_1}^\alpha, E_{f_2}^\alpha v_{f_2}^\alpha + E_{m_2}^\alpha v_{m_2}^\alpha\right] \end{aligned} \tag{17}$$

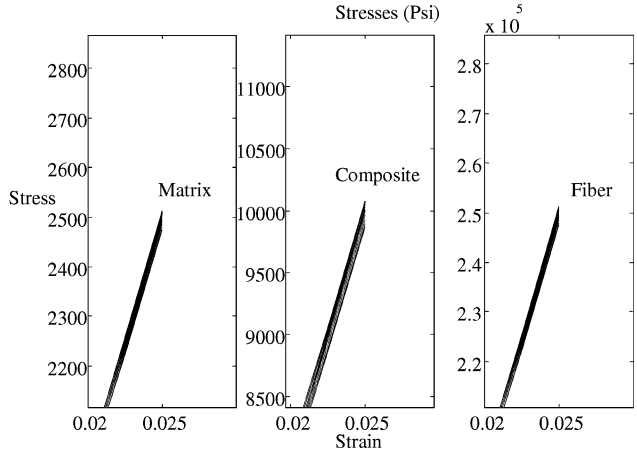

Fig. 5 Stresses in a continuous parallel fiber lamina under longitudinal tensile loading.

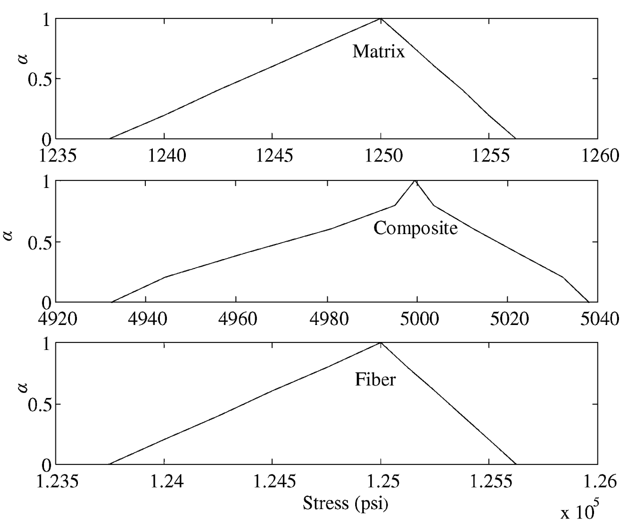

Fig. 6 Membership functions of the stresses for ε = 0.025.

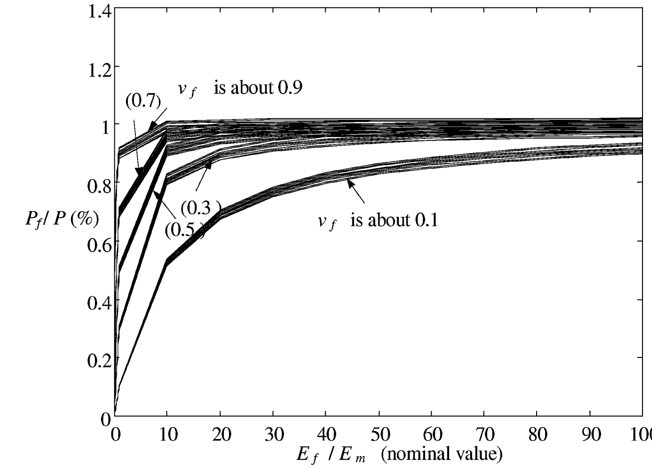

Fig. 7 Fraction of load shared by fibers in a continuous parallel fiber lamina in longitudinal tension.

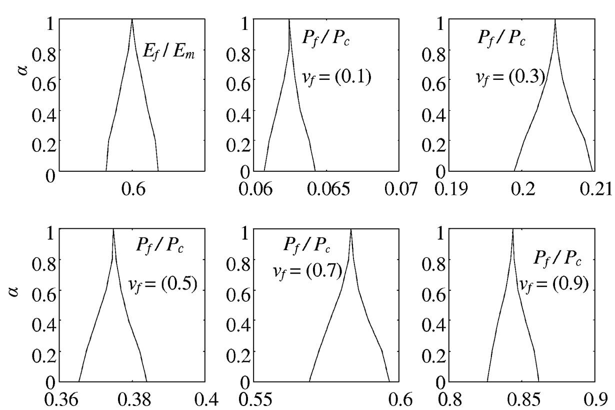

Fig. 8 Membership functions of P_f/P_c when E_f/E_m is (0.6) for different values of v_f.

where $E_{L_\alpha} = \{x | \mu_{E_L}(x) \geq \alpha\}$ and E_L is the longitudinal modulus of the composite. Equation (17) shows that the composite modulus lies in between the fiber and matrix moduli.

The fraction of load carried by the fibers in a unidirectional continuous fiber lamina is given by

$$(P_f/P)_\alpha = E_{f_\alpha}(\cdot)v_{f_\alpha}(/)\left\{E_{f_\alpha}(\cdot)v_{f_\alpha}(+)E_{m_\alpha}(\cdot)v_{m_\alpha}\right\}$$

$$= \left[\frac{E^\alpha_{f_1}v^\alpha_{f_1}}{E^\alpha_{f_2}v^\alpha_{f_2} + E^\alpha_{m_2}v^\alpha_{m_2}}, \frac{E^\alpha_{f_2}v^\alpha_{f_2}}{E^\alpha_{f_1}v^\alpha_{f_1} + E^\alpha_{m_1}v^\alpha_{m_1}}\right]$$

$$E^\alpha_{f_1}v^\alpha_{f_1} + E^\alpha_{m_1}v^\alpha_{m_1} > 0, \qquad \forall\alpha \in [0, 1] \quad (18)$$

Figure 7 shows the variation of (P_f/P) as a function of the ratio E_f/E_m and v_f. In Fig. 7, the nominal values (values when $\alpha = 1$) of E_f/E_m are used in plotting by choosing the fuzzy values of v_f as (0.1), (0.3), (0.5), (0.7), and (0.9). From Fig. 7, in polymeric matrix composites, E_f/E_m can be seen to be larger than the fuzzy number (10); thus, even for v_f of about (0.3), fibers can carry more than 80% of the composite load. Note that the fraction of the load carried by fibers can be increased by increasing the value of v_f, which in turn increases the total load carried by the composite. This feature is valid even in the absence of fuzzy uncertainties.

Figure 8 shows the membership functions of the ratio (P_f/P_c) corresponding to different values of v_f when $(E_f/E_c) = (0.6)$.

B. Discontinuous Parallel Fibers

The tensile load applied to a discontinuous fiber lamina is transferred to the fibers by a shearing mechanism between the fibers and the matrix. Because the matrix has a lower modulus, the longitudinal strain in the matrix is higher than that in the adjacent fibers. For a given fiber diameter and fiber/matrix interfacial condition, a critical fiber length l_c is calculated as

$$l_{c_\alpha} = \left(\sigma_{\mathrm{fu}_\alpha}(\cdot)d_{f_\alpha}\right)(/)\left(2(\cdot)\tau_{i_\alpha}\right)$$

$$= \left[\frac{\sigma^\alpha_{\mathrm{fu}_1}d^\alpha_{f_1}}{2\tau^\alpha_{i_2}}, \frac{\sigma^\alpha_{\mathrm{fu}_2}d^\alpha_{f_2}}{2\tau^\alpha_{i_1}}\right], \qquad \tau^\alpha_{i_1} > 0 \quad (19)$$

where σ_{fu} is the ultimate fiber strength, d_f is the fiber diameter, l_c is the minimum fiber length required for the maximum fiber stress to be equal to the ultimate fiber strength at its midlength, andτ_i is the shear strength of the fiber–matrix interface or the shear strength of the matrix adjacent to the interface, whichever is less. For fiber length larger than the minimum value l_c, that is, $l_f > l_c$, the longitudinal tensile strength of a unidirectional discontinuous fiber composite is given by

$$\sigma_{\mathrm{ltu}_\alpha} = \sigma_{\mathrm{fu}_\alpha}(\cdot)\left\{\left[2(\cdot)l_{f_\alpha}(-)l_{c_\alpha}\right](/)\left[2(\cdot)l_{f_\alpha}\right]\right\}(\cdot)v_{f_\alpha}(+)\sigma'_{m_\alpha}(\cdot)v_{m_\alpha}$$

$$= \left[\frac{\sigma^\alpha_{\mathrm{fu}_1}\left(2l^\alpha_{f_1} - l^\alpha_{c_2}\right)v^\alpha_{f_1}}{2l^\alpha_{f_2}} + \sigma'^\alpha_{m1}v^\alpha_{m_1}, \frac{\sigma^\alpha_{\mathrm{fu}_2}\left(2l^\alpha_{f_2} - l^\alpha_{c_1}\right)v^\alpha_{f_2}}{2l^\alpha_{f_1}} + \sigma'^\alpha_{m2}v^\alpha_{m_2}\right]$$

$$l^\alpha_{f_1} > 0 \quad (20)$$

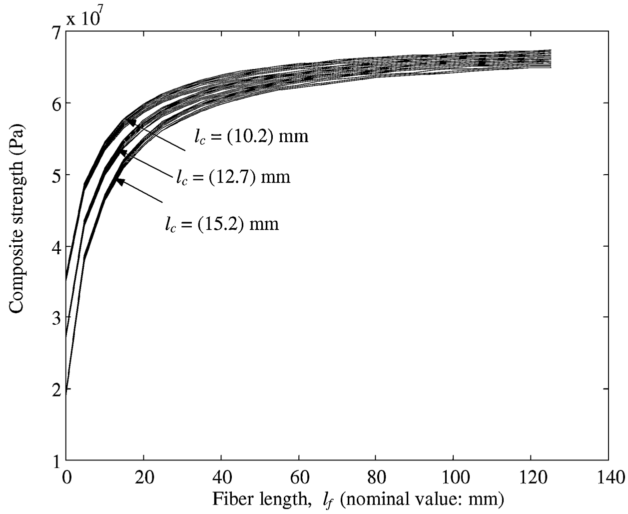

Fig. 9 Variation of the longitudinal strength of a discontinuous parallel fiber composite with fiber length.

where σ_{ltu} is the longitudinal tensile strength and σ'_m is the stress in the matrix at the instance of fiber failure. Equation (20) is shown graphically in Fig. 9 for three different values of l_c.

In Eq. (20), it is assumed that all fibers fail at the same strength level of σ_{fu}. It can be seen from Eq. (20) that discontinuous fibers always strengthen a matrix to a lesser degree than continuous fibers. However, for $l_f > 5l_c$, more than 90% strengthening can be achieved even with discontinuous fibers.

When the matrix is in the elastic state and the fiber–matrix bond is still unbroken, the interfacial shear stress does not remain constant but varies along the length of the fiber. When it is assumed that the matrix has the same strain as the composite, the following expressions can be derived for the fiber stress and the shear stress distributions along the length of a discontinuous fiber:

$$\sigma_{f_\alpha} = \left[E_{f_\alpha}(\cdot)\varepsilon_{1_\alpha}\right](-)E_{f_\alpha}(\cdot)\varepsilon_{1_\alpha}(\cdot)\cosh\left\{0.5(\cdot)\beta_\alpha(\cdot)l_{f_\alpha}(-)\beta_\alpha(\cdot)x_\alpha)\right\}$$
$$(/)\ \cosh\left\{0.5(\cdot)\beta_\alpha(\cdot)l_{f_\alpha}\right\} \qquad (21)$$

$$\tau_\alpha = 0.5(\cdot)E_{f_\alpha}(\cdot)\varepsilon_{1_\alpha}(\cdot)\beta_\alpha(\cdot)r_{f_\alpha}(\cdot)\sinh\left\{0.5(\cdot)\beta_\alpha(\cdot)l_{f_\alpha}(-)\beta_\alpha(\cdot)x_\alpha)\right\}$$
$$(/)\ \cosh\left\{0.5(\cdot)\beta_\alpha(\cdot)l_{f_\alpha}\right\} \qquad (22)$$

where σ_f is the longitudinal fiber stress at a distance x from its end, E_f is the fiber modulus, ε_1 is the composite longitudinal strain, r_f is the fiber radius, β is a fuzzy constant, and $(0) \le 2(\cdot)x \le l_f$. From Eqs. (21) and (22), the variation of the tensile stress in the fiber and the interfacial shear stress with x can be determined.

V. Transverse Tensile Loading

When a transverse tensile load is applied to the lamina, the fibers act as hard inclusions in the matrix instead of as load-carrying members. A simple equation for predicting the transverse tensile strength of a unidirectional lamina is

$$\sigma_{Ttu_\alpha} = \sigma_{mu_\alpha}(/)K_{\sigma_\alpha} = \left[\sigma^\alpha_{mu_1}\big/K^\alpha_{\sigma_2}, \sigma^\alpha_{mu_2}\big/K^\alpha_{\sigma_1}\right], \qquad K^\alpha_{\sigma_1} > 0 \qquad (23)$$

where

$$K_{\sigma_\alpha} = \left\{(1)(-)v_{f_\alpha}(\cdot)\left[(1)(-)E_{m_\alpha}(/)E_{f_\alpha}\right]\right\}$$
$$(/)\ \left\{(1)(-)\left[(4/\pi)(\cdot)v_{f_\alpha}\right]^{\frac{1}{2}}(\cdot)\left[(1)(-)E_{m_\alpha}(/)E_{f_\alpha}\right]\right\}$$
$$\cdot\left[(4/\pi)(\cdot)v_{f_\alpha}\right]^{\frac{1}{2}} = \left[\left(4v^\alpha_{f_1}\big/\pi\right)^{\frac{1}{2}}, \left(4v^\alpha_{f_2}\big/\pi\right)^{\frac{1}{2}}\right] \qquad (24)$$

Equation (23) assumes that the transverse strength of the lamina is limited by the ultimate tensile strength of the matrix, whereas K_σ is the maximum stress concentration in the matrix in which fibers are arranged in a square array.

VI. Longitudinal Compressive Loading

An important function of the matrix in a fiber-reinforced composite material is to provide lateral support and stability for fibers under longitudinal compressive loading. Because the modulus of the matrix is relatively low in polymeric matrix composites, a critical failure mode in longitudinal compression is fiber microbuckling. Extensional and shear modes are the two possible fiber microbuckling modes. The compressive strength in shear mode can be expressed as

$$\sigma_{\mathrm{lcu}_\alpha} = G_{m_\alpha}(/)v_{m_\alpha} = \left[G^\alpha_{m_1}\big/v^\alpha_{m_2}, G^\alpha_{m_2}\big/v^\alpha_{m_1}\right], \qquad v^\alpha_{m_1} > 0 \qquad (25)$$

where G_m is the shear modulus and v_m is the volume fraction of the matrix.

VII. Characteristics of a Fiber-Reinforced Lamina

A. Stress Transformation

In an orthotropic lamina with a fiber orientation angle θ, the stresses along the fiber orientation direction are different from those along the xy directions. The xy coordinate system, in general, does not coincide with the material coordinate system. Furthermore, composite laminates have several layers, each with different orientation of their material coordinates with respect to the laminate coordinates. Thus, there is a need to establish transformation relations between coordinate systems for stresses and strains. For stress transformation, two right-handed coordinate systems, namely, the 1–2–z system (material coordinate system) and the x–y–z system (laminate coordinate system) are defined with both 1–2 and x–y axes in the plane of the lamina and the z axis normal to the plane of the lamina. In the 1–2–z system, axis 1 is taken along the fiber length and represents the longitudinal direction of the lamina, and axis 2 is chosen normal to the fiber length and represents the transverse direction of the lamina. Together axes 1 and 2 constitute the principal material directions in the plane of the lamina. In the xyz system, x and y axes represent the loading directions.

In the stress analysis of an orthotropic lamina with a fiber orientation angle θ (angle between the axes 1 and x), the stress transformation equations are given by

$$\sigma_{11_\alpha} = \sigma_{xx_\alpha}(\cdot)[\cos(\theta_\alpha)]^2(+)\sigma_{yy_\alpha}(\cdot)[\sin(\theta_\alpha)]^2$$
$$(+)\ 2(\cdot)\tau_{xy_\alpha}(\cdot)\cos(\theta_\alpha)(\cdot)\sin(\theta_\alpha)$$

$$\sigma_{22_\alpha} = \sigma_{xx_\alpha}(\cdot)[\sin(\theta_\alpha)]^2(+)\sigma_{yy_\alpha}(\cdot)[\cos(\theta_\alpha)]^2$$
$$(-)\ 2(\cdot)\tau_{xy_\alpha}(\cdot)\cos(\theta_\alpha)(\cdot)\sin(\theta_\alpha)$$

$$\tau_{12_\alpha} = \left[\sigma_{yy_\alpha}(-)\sigma_{xx_\alpha}\right](\cdot)\sin(\theta_\alpha)(\cdot)\cos(\theta_\alpha)$$
$$(+)\ \tau_{xy_\alpha}(\cdot)\left[(\cos(\theta_\alpha))^2(-)(\sin(\theta_\alpha))^2\right] \qquad (26)$$

where $\cos(\theta_\alpha) = [\cos(\theta^\alpha_2), \cos(\theta^\alpha_1)]$, $\sin(\theta_\alpha) = [\sin(\theta^\alpha_1), \sin(\theta^\alpha_2)]$, $0 \le \theta^\alpha_1 \le \theta^\alpha_2 \le \pi/2$, and $(x_\alpha)^2 = [(x^\alpha_1)^2, (x^\alpha_2)^2]$.

B. Elastic Properties of a Lamina

1. *Unidirectional Continuous Fiber (0-Degree) Lamina*

The elastic properties of a unidirectional continuous fiber (0 deg) lamina can be calculated using standard relations. For example, the longitudinal modulus and the major Poisson's ration can be determined as[7]

$$E_{11_\alpha} = E_{f_\alpha}(\cdot)v_{f_\alpha}(+)E_{m_\alpha}(\cdot)v_{m_\alpha} \qquad (27)$$

$$\nu_{12_\alpha} = \nu_{f_\alpha}(\cdot)v_{f_\alpha}(+)\nu_{m_\alpha}(\cdot)v_{m_\alpha} \qquad (28)$$

Equations (27) and (28), as well as the other relations, can be derived using a simple mechanics of materials approach along with

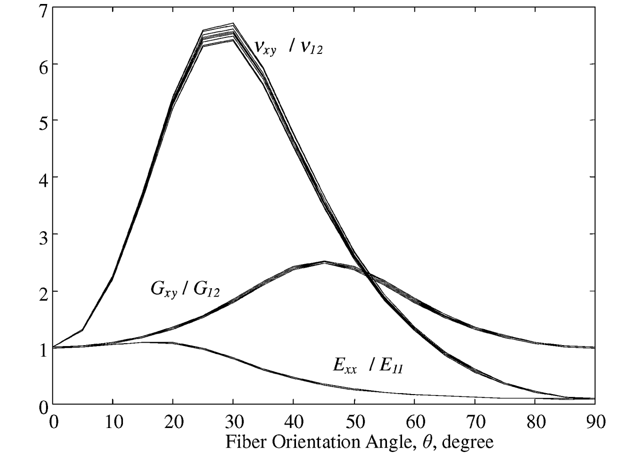

Fig. 10 Variation of elastic constants of a continuous fiber lamina with fiber orientation angle.

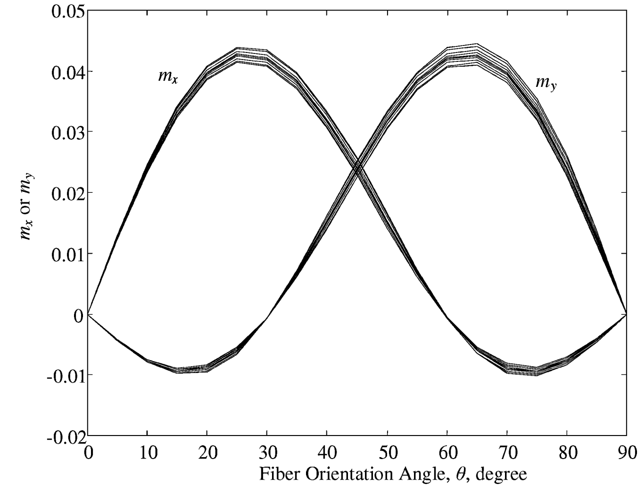

Fig. 11 Variations of coefficients of mutual influence with fiber orientation angle.

the following assumptions:

1) Both fibers and matrix are linearly elastic isotropic materials.
2) Fibers are uniformly distributed in the matrix.
3) Fibers are perfectly aligned.
4) Perfect bonding exists between fibers and matrix.
5) Composite lamina is free of voids.

2. Unidirectional Continuous Fiber Angle-Ply Lamina

The elastic properties of an angle-ply lamina in which continuous fibers are aligned at an angle θ from the positive x direction can be computed from standard crisp relations.[7] Figure 10 shows the variations of E_{xx}/E_{11}, G_{xy}/G_{12}, and ν_{xy}/ν_{12} as functions of fiber orientation angle θ for an angle-ply lamina.

VIII. Coefficients of Linear Thermal Expansion

For a unidirectional continuous fiber lamina, the coefficients of linear thermal expansion in the (0 deg) and (90 deg) directions, $\alpha_{11\alpha}$ and $\alpha_{22\alpha}$, can be calculated using the standard crisp relations,[7] where α_{fl} is the coefficient of linear thermal expansion of the fiber in the longitudinal direction, α_{fr} is the coefficient of linear thermal expansion of the fiber in the radial direction of the fiber section, and α_m is the coefficient of linear thermal expansion of the matrix. If the fibers are at an angle θ from the x direction, the coefficients of thermal expansion in the x and y directions can be calculated using α_{11} and α_{22} as

$$\alpha_{xx_\alpha} = \alpha_{11_\alpha}(\cdot)[\cos(\theta_\alpha)]^2(+)\alpha_{22_\alpha}(\cdot)[\sin(\theta_\alpha)]^2$$

$$\alpha_{yy_\alpha} = \alpha_{11_\alpha}(\cdot)[\sin(\theta_\alpha)]^2(+)\alpha_{22_\alpha}(\cdot)[\cos(\theta_\alpha)]^2$$

$$\alpha_{xy_\alpha} = 2(\cdot)\left[\alpha_{11_\alpha}(-)\alpha_{22_\alpha}\right](\cdot)\sin(\theta_\alpha)(\cdot)\cos(\theta_\alpha) \tag{29}$$

where α_{xx} and α_{yy} are the coefficients of linear expansion and α_{xy} is the coefficient of shear expansion. Observe that unless $\theta = (0\text{ deg})$ or (90 deg), a change in the temperature produces a shear strain because of the presence of α_{xy}. The other two coefficients, α_{xx} and α_{yy}, produce extensional strains in the x and y directions, respectively.

IX. Stress–Strain Relationships for a Thin Orthotropic Lamina

When fuzziness is introduced into the composite material parameters, the expressions denoting the stress–strain relationships will be slightly different from the crisp (or deterministic) ones. For a thin orthotropic lamina in plane stress ($\sigma_{zz} = \tau_{xz} = \tau_{yz} = (0)$), the strain–stress relations in the elastic range are given by

$$\varepsilon_{xx_\alpha} = \left[\sigma_{xx_\alpha}(/)E_{xx_\alpha}\right](-)\left[\nu_{yx_\alpha}(\cdot)\sigma_{yy_\alpha}(/)E_{yy_\alpha}\right](-)\left[m_{x_\alpha}(\cdot)\tau_{xy_\alpha}\right] \tag{30}$$

$$\varepsilon_{yy_\alpha} = \left[\sigma_{yy}(/)E_{yy_\alpha}\right](-)\left[\nu_{xy_\alpha}(\cdot)\sigma_{xx_\alpha}(/)E_{xx_\alpha}\right](-)\left[m_{y_\alpha}(\cdot)\tau_{xy_\alpha}\right] \tag{31}$$

$$\gamma_{xy_\alpha} = \left[\tau_{xy_\alpha}(/)G_\alpha\right](-)\left[m_{x_\alpha}(\cdot)\sigma_{xx_\alpha}\right](-)\left[m_{y_\alpha}(\cdot)\sigma_{yy_\alpha}\right] \tag{32}$$

where E_{xx}, E_{yy}, G_{xy}, and ν_{xy} are the elastic constants of the lamina obtained with fuzzy information and m_x and m_y are given by

$$m_{x_\alpha} = [\sin 2(\cdot)\theta_\alpha)](\cdot)\Big\{\left[\nu_{12_\alpha}(/)E_{11_\alpha}\right](+)\left[(1)(/)E_{22_\alpha}\right](-) \left(\tfrac{1}{2}\right)(/)G_{12_\alpha}(-)[\cos(\theta_\alpha)]^2(\cdot)\left[(1)(/)E_{11_\alpha}(+)2(\cdot)\nu_{12_\alpha}(/)E_{11_\alpha} (+)\,(1)(/)E_{22_\alpha}(-)(1)(/)G_{12_\alpha}\right]\Big\} \tag{33}$$

$$m_{y_\alpha} = [\sin 2(\cdot)\theta_\alpha)](\cdot)\Big\{\left[\nu_{12_\alpha}(/)E_{11_\alpha}\right](+)\left[(1)(/)E_{22_\alpha}\right] (-)\left(\tfrac{1}{2}\right)(/)G_{12_\alpha}(-)[\sin(\theta_\alpha)]^2(\cdot)[(1)(/)E_{11_\alpha}(+)2(\cdot)\nu_{12_\alpha} (/)\,E_{11_\alpha}(+)(1)(/)E_{22_\alpha}(-)(1)(/)G_{12_\alpha}\big]\Big\} \tag{34}$$

The new elastic constants m_x and m_y represent the influence of shear stresses on the extensional strains in Eqs. (30) and (31) and the influence of normal stresses on the shear strain in Eq. (32). These constants are called the coefficients of mutual influence. Both m_x and m_y are functions of the fiber orientation angle θ, and Fig. 11 shows the relationships.

Unlike isotropic lamina, the extensional and shear deformations are coupled in a general orthotropic lamina, that is, the normal stresses cause both normal strains and shear strains and the shear stress causes both shear strain and normal strains. For $\theta = (0\text{ deg})$ or (90 deg), both m_x and m_y are zero, and therefore, for these fiber orientations there is no extension–shear coupling. Such a lamina, in which the principal material axes (1 and 2) coincide with the loading axes (x and y), is said to be specially orthotropic. For a specially orthotropic lamina, the strain–stress relations are given by

$$\varepsilon_{xx_\alpha} = \varepsilon_{11_\alpha} = \left[\sigma_{xx_\alpha}(/)E_{11_\alpha}\right](-)\left[\nu_{21_\alpha}(\cdot)\sigma_{yy_\alpha}(/)E_{22_\alpha}\right]$$

$$\varepsilon_{yy_\alpha} = \varepsilon_{22_\alpha} = \left[\sigma_{yy}(/)E_{22_\alpha}\right](-)\left[\nu_{12_\alpha}(\cdot)\sigma_{xx_\alpha}(/)E_{11_\alpha}\right]$$

$$\gamma_{xy_\alpha} = \gamma_{yx_\alpha} = \gamma_{12_\alpha} = \gamma_{21_\alpha} = \left[\tau_{xy_\alpha}(/)G_{12_\alpha}\right] \tag{35}$$

X. Applications and Discussion

To illustrate the application of fuzzy analysis of composite material mechanics, several numerical applications are considered. The uncertainties of all of the parameters are assumed to lie in a small range [−1%, +0.5%] of their respective deterministic values and the membership functions are assumed to be of triangular form.

Table 2 Numerical solution of application 2

α	σ_{xx}, Pa	θ, deg	σ_{11}, Pa	σ_{22}, Pa	τ_{12}, Pa
0.0	9.90000×10^6	−29.70	7.42949×10^6	2.44811×10^6	4.37151×10^6
0.2	9.92000×10^6	−29.76	7.44025×10^6	2.45612×10^6	4.36456×10^6
0.4	9.94000×10^6	−29.82	7.45827×10^6	2.46945×10^6	4.35393×10^6
0.6	9.96000×10^6	−29.88	7.47923×10^6	2.48351×10^6	4.34332×10^6
0.8	9.98000×10^6	−29.94	7.49455×10^6	2.49538×10^6	4.33386×10^6
1.0	1.00000×10^7	−30.00	7.50000×10^6	2.50000×10^6	4.33013×10^6
0.8	1.00100×10^7	−30.03	7.50633×10^6	2.50298×10^6	4.32324×10^6
0.6	1.00200×10^7	−30.06	7.51935×10^6	2.50869×10^6	4.30874×10^6
0.4	1.00300×10^7	−30.09	7.53624×10^6	2.51539×10^6	4.29120×10^6
0.2	1.00400×10^7	−30.12	7.55187×10^6	2.52217×10^6	4.27135×10^6
0.0	1.00500×10^7	−30.15	7.56479×10^6	2.52618×10^6	4.25826×10^6

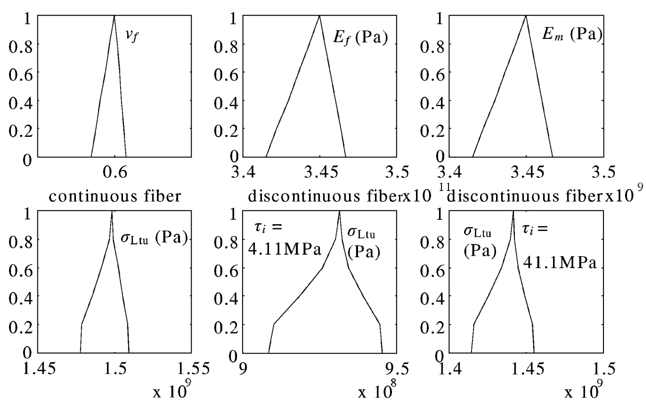

Fig. 12 Membership functions of fuzzy variables in application 1: horizontal axis denotes value of the parameter, and vertical axis denotes value of the membership function.

A. Application 1

A unidirectional fiber composite contains about (60) vol% of HMS-4 carbon fibers in an epoxy matrix. E_f is about (345) GPa, E_m is about (3.45) GPa, σ_{fu} is about (2.48) GPa, and σ_{my} is about (138) MPa, and the longitudinal tensile strength of the composite is to be determined for the following cases:

1) The fibers are all continuous.

2) The fibers are (3.17) mm long and τ_i is either about (4.11) MPa or (41.1) MPa.

The fiber failure strain is given by $\varepsilon_{\mathrm{fu}_\alpha} = \sigma_{\mathrm{fu}_\alpha}(/)E_{f_\alpha}$ and the matrix yield strain by $\varepsilon_{my_\alpha} = \sigma_{my_\alpha}(/)\ E_{m_\alpha}$. The stress in the matrix at the instance of fiber failure can be represented as $\sigma'_{m_\alpha} = E_{m_\alpha}(\cdot)\varepsilon_{fu_\alpha}$. For case 2; with (4.11) MPa, one form of Eq. (15) can be used as $\sigma_{\mathrm{ltu}_\alpha} = \sigma_{\mathrm{fu}_\alpha}(\cdot)v_{f_\alpha}(+)\sigma'_{m_\alpha}(\cdot)v_{m_\alpha}$. For case 2 with (41.1) MPa, $l_{c_\alpha} = \sigma_{\mathrm{fu}_\alpha}(\cdot)d_{f_\alpha}\ (/)\{2(\cdot)\tau_{i_\alpha}\}$ is used to compute l_c. Because $l_f > l_c$, Eq. (20) is used to find the solution for different values of τ_i. Figure 12 shows the graphical representation of the fuzzy variables. It can be seen that, although the input variables are assumed to follow triangular membership functions, the output parameters exhibit nonlinear behavior for the membership functions with the widths of uncertainty (corresponding to $\alpha = 0$) different for different parameters.

B. Application 2

A normal stress σ_{xx} of about (10) MPa is applied to a unidirectional angle-ply lamina containing fibers at (30 deg) to the x axis, implying that $\theta = -30$ deg. Determine the stresses in the principal material directions.

Because $\sigma_{yy} = \tau_{xy} = (0)$, from Eq. (26), the stresses can be computed to obtain the results shown in Table 2. Figure 13 shows the membership functions implied by the results of Table 2. These results also indicate that the membership functions of the output parameters are nonlinear for linear membership functions of the input variables. The ranges of uncertainties (corresponding to $\alpha = 0$) are found to be 1.81, 3.12, and 2.63% for σ_{11}, σ_{22}, and τ_{12}, respectively.

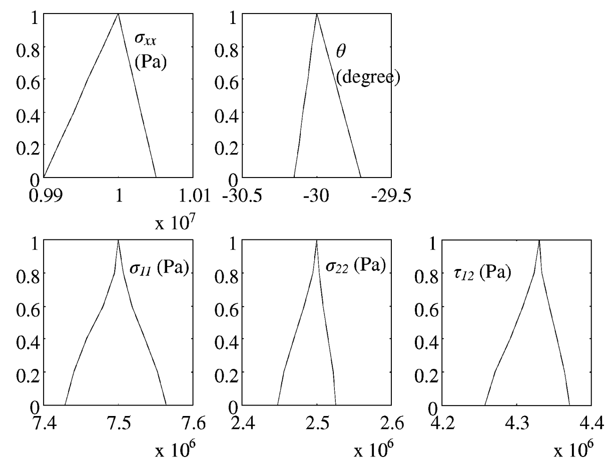

Fig. 13 Membership functions of fuzzy variables in application 2: horizontal axis denotes value of the parameter, and vertical axis denotes value of the membership function.

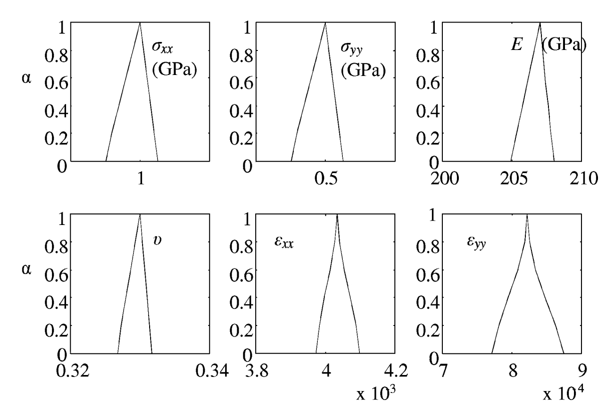

Fig. 14 Membership functions of application 3, case 1: horizontal axis denotes value of the parameter, and vertical axis denotes value of the membership function.

C. Application 3

A thin plate is subjected to a biaxial stress field with σ_{xx} of about (1) GPa and σ_{yy} of about (0.5) GPa. Calculate the strains in the xy directions if the plate is made of 1) steel, 2) a (0 deg) unidirectional boron–epoxy composite, and 3) a (45 deg) unidirectional boron–epoxy composite.

In the case of steel, E is about (207) GPa, and ν is about (0.33). The fuzzy form of isotropic relations are used to find the strains. In case 2, for the (0 deg) unidirectional boron–epoxy composite, E_{11} is about (207) GPa, E_{22} is about (19) GPa, ν_{12} is about (0.21), and G_{12} is about (6.4) GPa. Using Eq. (35), we can obtain the desired results. In case 3, with a (45-deg) unidirectional boron–epoxy composite, Eqs. (30–34) are used to get the required results. The results are shown in Figs. 14–16. From these results, it is observed that the

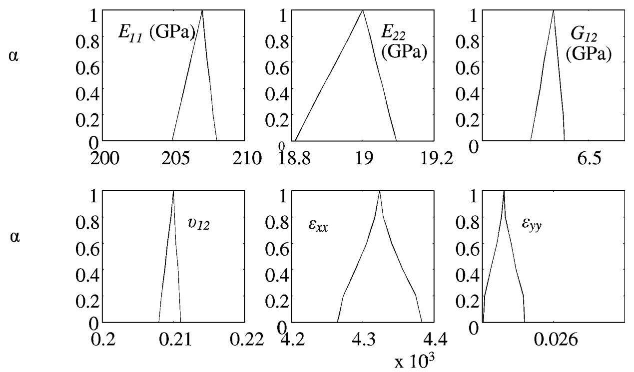

Fig. 15 Membership functions of application 3, case 2: horizontal axis denotes value of the parameter, and vertical axis denotes value of the membership function.

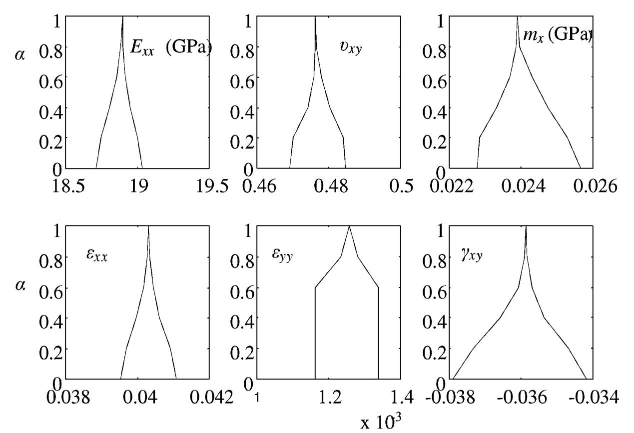

Fig. 16 Membership functions of application 3, case 3: horizontal axis denotes value of the parameter, and vertical axis denotes value of the membership function.

nature of membership functions of the output parameters varies with the material and the orientation of the fibers. In addition to variations in the nonlinearities and ranges, variations are also observed in the nominal values of the output parameters in the three cases (for the same stress conditions).

XI. Conclusions

The fuzzy analysis of fiber–matrix interactions in a unidirectional lamina and the characteristics of a fiber-reinforced lamina are presented. From the analysis and computations presented, the following observations can be made: 1) For the triangular form of membership functions of input parameters, the membership functions of the computed results may not remain triangular; they may have a distorted form. 2) The various characteristics of the fuzzy composite material will change within certain ranges (not just deterministic values), and for each range, we can find the interval of confidence for each α presumption. 3)The characteristics of a lamina with input parameters described in linguistic terms can also be determined using the fuzzy approach. 4) The computed results will involve much larger deviations than those of the inputs. The deviation of a fuzzy number is defined as

$$\text{deviation} = \frac{(\text{value at}\,\alpha = 0^+) - (\text{value at}\,\alpha = 0^-)}{\text{nominal value of the fuzzy number}}$$

For an input deviation of about 1%, the deviation of the results may be 5–10% or even larger. This is a reflection of the characteristic of fuzzy computations. Each computational step with fuzzy numbers will result in a wider range than the previous step. Therefore, some compression and truncation methods must be considered to make the final result reasonable and more accurate when larger uncertainties are involved. Despite the fact that the input data or information is fuzzy, most of the actions or decisions to be implemented are expected to be crisp in practical applications. Although not presented in this work, a defuzzification procedure can be used to convert a fuzzy quantity to an equivalent crisp or precise number. The maximum-membership principle and the centroid method are commonly used for defuzzification.[1]

When fuzziness is introduced into mechanics equations, the computations will yield more realistic results that reflect the characteristics of the composite materials realized in practical applications. This work represents the first attempt at applying fuzzy theory to composite material mechanics. The fuzzy approach presented in this work, based on α cuts, can be considered to be a generalization of the interval and the probabilistic methods. Considerable work needs to be done in this area, and future work involves the consideration of fuzziness in the modeling, analyses, and design of composite structures. The fuzzy analysis can also be extended to structural control involving composite materials.

References

[1]Kaufmann, A., and Gupta, M. M., *Introduction to Fuzzy Arithmetic Theory and Application*, International Thomson Computer Press, Boston, 1991.

[2]Wanthal, S. P., and Petter, E. H., "Probabilistic Assessment of Composite Constituent, Lamina and Laminate Properties," *Proceedings of the Structures, Structural Dynamics, and Materials Conference*, AIAA, Washington, DC, 1993, pp. 1218–1227.

[3]Matthews, F. L., and Rawlings, R. D., *Composite Materials: Engineering and Science*, Chapman and Hall, London, 1994, Chap. 2.

[4]Civanlar, M. R., and Trussell, H. J., "Constructing Membership Functions Using Statistical Data," *Fuzzy Sets and Systems*, Vol. 18, 1986, pp. 1–13.

[5]Rao, S. S., and Sawyer, J. P., "Fuzzy Finite Element Approach for the Analysis of Imprecisely Defined Systems," *AIAA Journal*, Vol. 33, No. 12, 1995, pp. 2364–2370.

[6]Rao, S. S., and Berke L., "Analysis of Uncertain Structural Systems Using Interval Analysis," *AIAA Journal*, Vol. 35, No. 4, 1997, pp. 727–735.

[7]Mallick, P. K., *Fiber-Reinforced Composites: Materials, Manufacturing, and Design*, Marcel Dekker, New York, 1993, Chaps. 1–4.

K. N. Shivakumar
Associate Editor